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Estimation of partial linear error-in-variables models for ρ^{-}-mixing dependence data

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Consider the partly linear regression model $Y = x\beta + g(t) + e$ where the explanatory x is erroneously measured, and both t and the response Y are measured exactly, the random error e is ρ^{-}-mixing. Let $\tilde{x}$ be a surrogate variable observed instead of the true x in the primary survey data. Assume that in addition to the primary data set containing N observations of $\left\{(Y_j, \tilde{x}_j, t_j)_{j=n+1}^{n+N}\right\}$, which is ρ^{-}-mixing data sets, an independent validation data containing n observations of $\left\{(x_j, \tilde{x}_j, t_j)_{j=1}^{n}\right\}$ is available. The exact observations on x may be obtained by some expensive or diffcult procedures for only a small subset of subjects enrolled in the study. In this paper, inspired by Berberan-Santos et al. [J. Math. Chem. 37 (2005)101], a semiparametric method with the primary data is employed to obtain the estimators of β and $g(\cdot)$ based on the least squares criterion with the help of validata. The proposed estimators are proved to be strongly consistent.

KEY WORDS: ρ^{-}-mixing, partial linear model, validation data, strong consistency, ρ^{*}-mixing, negatively associated

AMS subject classification: 62J02, 62G05

1. Introduction

Consider the partly linear regression model

$$Y = x\beta + g(t) + e \tag{1.1}$$

where $(x, t) \in R \times [0, 1]$ are the norandom design points, β is an unknown parameter, $g(\cdot)$ is an unknown smooth function defined [0, 1] and $e = Y - x\beta - g(t)$ are the random errors, $Ee = 0$ and $Ee^2 < \infty$.

If the $x$is mismeasured, the obtained do not follow the model specified in (1.1). It is well known that ignoring the measurement error and naively performing a usual regression analysis may lead to relations and incorrect conclusions.

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Often economic variables such as earnings and work hours in the survey reports of the labor market are erroneously measureed. A validation data set in this case can be obtained from the administrative payroll records of employees' earnings and work hours. In the evaluation of the smoking behavior, self-report data are relatively inexpensive to ascertain but may be subject to error. Expensive chemical analysis of saliva samples for the presence of cotininecan be performed for a small subset of subjects enrolled in study to produce a more accurate measure of smoking behavior. More examples can be found in Peps [1].

The model (1.1) has been investigated widely in the literature over last decade. The asymptotics of the various estimators in the models (1.1) with independent errors have been studied in Gao et al. [2], Gin (1995), Wang [3] and the references therein.

An important case of the dependent error is the case when the errors are ρ^--mixing. Because ρ^--mixing sequence is more general than negatively associated (NA) sequence or ρ^*-mixing sequence. As for ρ^--mixingsequence, Zhang and Wanggave the concept of ρ^--mixing in 1999 (we can also see in ref. [4]).The concept of ρ^--mixing see the following definition.

Definition 1 *A field $\{X_i, i \in N^d\}$ is called ρ^--mixing if*

$$\rho^-(s) = \sup\{(\rho^-(S,T); S, T \subset N^d, dist(S,T) \geqslant s\} \to 0 (s \to \infty),$$

where

$$\rho^-(S,T) = 0 \vee \sup\left\{\frac{Cov\{f(X_i; i \in S), g(X_j; j \in T)\}}{[Var\{f(X_i; i \in S)\}Var\{g(X_j; j \in T)\}]^{1/2}}; f, g \in \mathcal{C}\right\}.$$

In 2005, Berberan-Santos, Bodunov, and Poglianil [5] have discussed the classical and the quantum mechanical description of a one-dimensional motion of a particle in the presence of a gravitational field. Their attention is centered on the evolution of classical and quantum mechanical position probability distribution function. The classical case has been compared with three different quantum cases: (a) a quantum stationary case, (b) a quantum non-stationary zero approximation case, where the wave packet has the shape of the first eigenfunction,and (c) a quantum non-stationary general case, where the wave packet is a superposition of stationary states.

The main purpose of this paper is to show how we can utilize the primary data to obtain a consistent estimator when there measurement errors in variables, with ρ^--mixing random error.

2. Estimation

Let $\tilde{x}$ be a surrogate variable observed instead of the true $x$in the primary survey data. Assume that in addition to the primary data set containing

N observations of $\{(Y_j, \tilde{x}_j, t_j)_{j=n+1}^{n+N}\}$, which is ρ^--mixing data sets, an independent validation data containing n observations of $\{(x_j, \tilde{x}_j, t_j)_{j=1}^{n}\}$ is available.

Inference based on surrogate data and validation samples with independent data sets, has been also the focus of much research. See, for example, Carroll and Wand [6], Peps [1], Sepanski and Carroll [7], Sepanski et al. [8], Sepanski and Lee [9], Taupin [10], and references therein. Measurement error has recently been shown by Newey [11] to be important application in the estimation of nonlinear Engel curves. In the following, the random error $e_i = Y_i - x_i\beta - g(t_i)$ is ρ^--mixing dependent.

By employing the validation data, the estimator of the true x_i can be defined as

$$u_n(v_i) = \frac{\sum_{j=1}^{n} x_j K_1((v_j - v_i)/b_n)}{\sum_{j=1}^{n} K_1((v_j - v_i)/b_n)}, i = n+1, \ldots, n+N \tag{2.1}$$

where $v_j = (\tilde{x_j}, t_j)$, $j = 1, \ldots, n+N$, $K_1(\cdot)$ is a two-dimensional kernel function and b_n is a bandwith tending to zero. Let

$$\omega_{nj}(t) = \frac{K_2((t - t_j)/h_n)}{\sum_{j=n+1}^{n+N} K_2((t - t_j)/h_n)}, j = n+1, \ldots, n+N \tag{2.2}$$

where $K_2(\cdot)$ is also a kernel function and h_n a bandwith tending to zero.

If β is known to be the true parameter in (1.1), then $Y_i - x_i\beta = g(t_i) + e_i$. Hence, the natural estimator of $g(\cdot)$ is

$$g_{nN}(t, \beta) = \sum_{i=n+1}^{n+N} \omega_{ni}(t)(Y_i - u_n(v_i)\beta) =: g_{1N}(t) - g_{2N}(t)\beta. \tag{2.3}$$

The least square estimator, say β_{nN}, of β is defined by

$$\sum_{j=n+1}^{n+N} (Y_i - u_n(v_i)\beta - g_{1N}(t_j) + g_{2N}(t_j)\beta)^2 = \min.$$

Solution to the equation

$$\sum_{j=n+1}^{n+N} [(u_n(v_j) - g_{2N}(t_j))(Y_j - g_{1N}(t_j) - [u_n(v_j)\beta - g_{2N}(t_j)]\beta)] = 0$$

we have

$$\beta_{nN} = \sum_{j=n+1}^{n+N} \bar{u}_n(v_j)\bar{Y}_j / S_{nN}^2 \tag{2.4}$$

where

$$\bar{u}_n(v_i) = u_n(v_i) - \sum_{j=n+1}^{n+N} \omega_{nj}(t_i)u_n(v_j) = u_n(v_i) - g_{2N}(t_i),$$

$$\bar{Y}_i = Y_i - \sum_{j=n+1}^{n+N} \omega_{ni}(t_i)Y_j = Y_i - g_{1N}(t_i), S_{nj}^2$$
$$= \sum_{i=n+1}^{n+j} [\bar{u}_n(v_i)]^2, i, j = n+1, \ldots, n+N.$$

(2.3), (2.4) together then yield the final estimator of$g(\cdot)$, as follows

$$\tilde{g}_{nN}(t) =: g_{1N}(t) - g_{2N}(t)\beta_{nN}. \tag{2.5}$$

Now the model (1.1) can be rewritten as

$$\begin{aligned} Y_j &= u_n(v_j)\beta + g(t_j) + \varepsilon_j \\ \varepsilon_j &= x_j\beta + e_j - u_n(v_j)\beta \end{aligned} \tag{2.6}$$

where $j = n+1, \ldots, n+N. e_j = Y_j - x_j\beta - g(t_j)$ is ρ^--mixing random errors, $\varepsilon_j = x_j\beta + e_j - u_n(v_j)\beta$, $\varepsilon_j - E\varepsilon_j = e_j$ and$\sup_{j\geqslant 1} E(\varepsilon_j - E\varepsilon_j)^2 = \sup_{j\geqslant 1} Ee_j^2 < \infty$.

3. The main theorems and the Proof

Throughout this paper, C will represent a positive constant though its value may change from one appearance to the next, and $a_n = O(b_n)$ will mean $a_n \leqslant Cb_n$. Let $v = (\tilde{x}, t)$, $\|v\| = |\tilde{x}| + |t|$.

In order to obtain the main results in this section, we shall need the following assumptions:

$(A1)$ $CI(\|v\| \leqslant C) \leqslant K_1(v) \leqslant C, CI(|t| \leqslant C) \leqslant K_2(v) \leqslant C$
$(A2)$ $\lim_{\|v\|\to\infty} K_1(v) = 0, \lim_{|t|\to\infty} K_2(t) = 0$
$(A3)$ $n^{-1}\sum_{i=1}^n |x_i| \leqslant C, N^{-1}\sum_{i=n+1}^{n+N} |x_i| \leqslant C$
$(A4)$ $\sum_{i=n+1}^{n+N} |\bar{u}_n(v_i)| \leqslant C\sum_{i=n+1}^{n+N}[\bar{u}_n(v_i)]^2$
$(A5)$ $\sum_{i=n+1}^{n+N}[\bar{u}_n(v_i)]^2 \to \infty$ as $N \to \infty$
$(A6)$ $g(t)$ is acontinuous function defined on the interval [0,1].

Theorem 1. In addition to (2.6), assumptions that (A1)–(A4), (A6) hold, then

$$E|\beta_{nN} - \beta|^2 = O(S_{nN}^{-2}) \tag{2.7}$$

$$E\left(\sup_{t\in[0,1]} |\tilde{g}_{nN}(t) - g(t)|^2\right) = O(S_{nN}^{-2}) + O(N^{-1}) \tag{2.8}$$

Theorem 2. In addition to (2.6), assumptions that (A1)–(A6), then

$$\lim_{n,N\to\infty} \beta_{nN} = \beta \text{ a.s.} \tag{2.9}$$

$$\lim_{n,N\to\infty} \sup_{t\in[0,1]} |\tilde{g}_{nN}(t) - g(t)| = 0 \ a.s. \tag{2.10}$$

In order to prove our results, we need the following lemmas.

Lemma 2.1. Let $\{X_i, i \geqslant 1\}$ be a ρ^--mixing sequence of random variables, $EX_i = 0$, $E|X_i|^p < \infty$ for some $p \geqslant 2$ and for every $i \geqslant 1$. Thenthere exists $C = C(p, \rho^-(n))$, such that

$$E \max_{1\leqslant k\leqslant n} |\sum_{i=1}^{k} X_i|^p \leqslant C \left\{ \left(E \max_{1\leqslant k\leqslant n} |\sum_{i=1}^{k} X_i| \right)^p + \sum_{i=1}^{n} E|X_i|^p + \left(\sum_{i=1}^{n} EX_i^2 \right)^{p/2} \right\}.$$

Proof of Lemma 2.1. The proof is similar to the proof of Lemma A.2. in Zhang and Wen (2001) with some change only,and so is omited here.

Lemma 2.2. Let $\{X_i, i \geqslant 1\}$ be a ρ^--mixing sequence of random variables, $EX_i = 0$, $E|X_i|^p < \infty$ for some $p \geqslant 2$ and for every $i \geqslant 1$. Then there exists $C = C(p, \rho^-(n))$, such that

$$E \max_{1\leqslant k\leqslant n} |\sum_{i=1}^{k} X_i|^p \leqslant C \left\{ \sum_{i=1}^{n} E|X_i|^p + \left(\sum_{i=1}^{n} EX_i^2 \right)^{p/2} \right\}.$$

Proof of Lemma 2.2. Using Lemma 2.1, the proof of Lemma 2.2 is similar to the proof of Theorem 2.1 in Utev and Peligrad [12].

Proof of Theorem 1. Proof of Theorem 1 directly follows from the proof of Theorem 2, hence, in the following we prove only Theorem 2.

Proof of Theorem 2. It is easy to see

$$\begin{aligned}
&\beta_{nN}-\beta\\
&=S_{nN}^{-2}\sum_{i=n+1}^{n+N}\bar{u}_n(v_i)\left([u_n(v_i)\beta+g(t_i)+\varepsilon_i]\right.\\
&\quad\left.-\sum_{j=n+1}^{n+N}\omega_{nj}(t_j)[u_n(v_j)\beta+g(t_j)+\varepsilon_j]\right)-\beta\\
&=S_{nN}^{-2}\sum_{i=n+1}^{n+N}\bar{u}_n(v_i)\varepsilon_i-S_{nN}^{-2}\sum_{j=n+1}^{n+N}\left(\sum_{i=n+1}^{n+N}\omega_{nj}(t_i)\bar{u}_n(v_j)\right)\varepsilon_j\\
&\quad+S_{nN}^{-2}\sum_{i=n+1}^{n+j}\bar{u}_n(v_i)\left(g(t_i)-\sum_{j=n+1}^{n+N}\omega_{nj}(t_j)g(t_j)\right)\\
&\quad+S_{nN}^{-2}\sum_{i=n+1}^{n+N}\bar{u}_n(v_i)\left(u_n(v_i)\beta-\sum_{j=n+1}^{n+N}\omega_{nj}(t_i)u_n(v_j)\beta\right)-\beta\\
&=S_{nN}^{-2}\sum_{j=n+1}^{n+N}\left(\bar{u}_n(v_j)-\sum_{i=n+1}^{n+N}\omega_{nj}(t_i)\bar{u}_n(v_i)\right)(\varepsilon_j-E\varepsilon_j)\\
&\quad+S_{nN}^{-2}\sum_{j=n+1}^{n+j}\left(\bar{u}_n(v_j)-\sum_{i=n+1}^{n+N}\omega_{nj}(t_i)\bar{u}_n(v_i)\right)(x_j-u_n(v_j))\beta\\
&\quad+S_{nN}^{-2}\sum_{i=n+1}^{n+N}\bar{u}_n(v_i)\left(g(t_i)-\sum_{j=n+1}^{n+N}\omega_{nj}(t_i)g(t_j)\right)\\
&=:I_1+I_2+I_3.
\end{aligned}\tag{2.11}$$

Notice that the uniform continuity of the function $g(\cdot)$on the interval [0,1] and the definition of (2.2), we have

$$\begin{aligned}
&\max_{n+1\leqslant i\leqslant n+N}\left(g(t_i)-\sum_{j=n+1}^{n+N}\omega_{nj}(t_i)g(t_j)\right)\\
&=\max_{n+1\leqslant i\leqslant n+N}\sum_{j=n+1}^{n+N}\omega_{nj}(t_i)(g(t_i)-g(t_j))(I(|t_i-t_j|>C)+I(|t_i-t_j|\leqslant C))
\end{aligned}$$

$$\leqslant \max_{n+1\leqslant i\leqslant n+N} 2\sup_t |g(t)| \sum_{j=n+1}^{n+N} \omega_{nj}(t_i)(I(|t_i - t_j| > C)) + o(1)$$
$$\times \max_{n+1\leqslant i\leqslant n+N} \sum_{j=n+1}^{n+N} \omega_{nj}(t_i)$$
$$\leqslant C \max_{n+1\leqslant i\leqslant n+N} \sum_{j=n+1}^{n+N} \omega_{nj}(t_i) I(|t_i - t_j| > C) + o(1).$$

From $h_n \to 0, (|t_i - t_j|/h_n)I(|t_i - t_j| > C) \to \infty, n \to \infty$and (A2), we have $K_2((t_i - t_j)/h_n)I(|t_i - t_j| > C) = o(1)$. Which, together with (A1), implies that

$$\max_{1\leqslant i\leqslant n} \sum_{j=n+1}^{n+N} \omega_{nj}(t_i)(I(|t_i - t_j| > C)) \leqslant \sum_{j=n+1}^{n+N} \frac{o(1)}{CN} = o(1). \tag{2.12}$$

It follows from assumption (A4) that

$$I_3 = S_{nN}^{-2} \sum_{i=n+1}^{n+N} \bar{u}_n(v_i) \left(g(t_i) - \sum_{j=n+1}^{n+N} \omega_{nj}(t_i) g(t_j) \right) = o(1).$$

By assumption (A1) and (A3), it is easy to obtain that

$$\max_{n+1\leqslant i,j\leqslant n+N} |\omega_{nj}(t_i)| \leqslant CN^{-1},$$
$$\max_{n+1\leqslant i\leqslant n+N} \sum_{j=n+1}^{n+N} |\omega_{nj}(t_i)| \leqslant C,$$
$$\max_{n+1\leqslant i\leqslant n+N} |\bar{u}_n(v_i)| \leqslant C \sum_{j=1}^{n} \frac{|x_j|}{n} \leqslant C.$$

Since assumption $(A5)$, $S_{nj}^2 \to \infty, S_{n(j+1)}^2/S_{nj}^2 = 1 + \frac{(\bar{u}_n(v_{j+1}))^2}{S_{nj}^2} \to 1$, as $j \to \infty$. There exists an increasing sequence of positive integers $\{j_k\}$ such that $S_{nj_k}^2 \sim 2^k$ as $k \to \infty$.
Now use the Lemma 2.2, we have

$$E\left(\max_{n+1\leqslant k\leqslant n+N} \sum_{i=n+1}^{k} \bar{u}_n(v_i)(\varepsilon_i - E\varepsilon_i) \right)^2 \leqslant C \sum_{i=n+1}^{n+N} (\bar{u}_n(v_i))^2 \sup_i Ee_i^2$$
$$= CS_{nN}^2 \tag{2.13}$$

and

$$E\left(\max_{n+1\leqslant k\leqslant n+N}\sum_{j=n+1}^{k}\left(\sum_{i=n+1}^{n+N}\omega_{nj}(t_i)\bar{u}_n(v_i)\right)(\varepsilon_j-E\varepsilon_j)\right)^2$$
$$\leqslant C\sum_{j=n+1}^{n+N}\left(\sum_{i=n+1}^{n+N}\omega_{nj}(t_i)\bar{u}_n(v_i)\right)^2\sup_i Ee_i^2$$
$$\leqslant C\sum_{j=n+1}^{n+N}\sum_{i=n+1}^{n+N}\omega_{nj}^2(t_i)\sum_{i=n+1}^{n+N}(\bar{u}_n(v_i))^2$$
$$\leqslant CS_{nN}^2 N\max_{i,j}|\omega_{nj}(t_i)|\max_i\sum_{j=n+1}^{n+N}|\omega_{nj}(t_i)|$$
$$\leqslant CS_{nN}^2. \tag{2.14}$$

Hence, for every N, there exists k, such that $j_k < N \leqslant j_{k+i}$, by Chebychev inequality, it follows that

$$P\left(|S_{nj_k}^{-2}\sum_{i=n+1}^{n+j_k}\bar{u}_n(v_i)(\varepsilon_j-E\varepsilon_j)|>\varepsilon\right)$$
$$\leqslant CS_{nj_k}^{-4}E\left(\sum_{i=n+1}^{n+j_k}\bar{u}_n(v_i)(\varepsilon_j-E\varepsilon_j)\right)^2\leqslant CS_{nj_k}^{-2}\leqslant C2^{-k}, \tag{2.15}$$

$$P\left(|S_{nj_k}^{-2}\max_{j_k<l\leqslant j_k+1}\sum_{i=n+j_k+1}^{n+l}\bar{u}_n(v_i)(\varepsilon_j-E\varepsilon_j)|>\varepsilon\right)$$
$$\leqslant CS_{nj_k}^{-4}E\left(\sum_{i=n+j_k+1}^{n+j_{k+1}}\bar{u}_n(v_i)(\varepsilon_j-E\varepsilon_j)\right)^2\leqslant CS_{nj_k}^{-4}S_{nj_{k+1}}^2\leqslant C2^{-k}, \tag{2.16}$$

$$P\left(|S_{nj_k}^{-2}\sum_{j=n+1}^{n+j_k}\left(\sum_{i=n+1}^{n+N}\omega_{nj}(t_i)\bar{u}_n(v_i)\right)(\varepsilon_i-E\varepsilon_i)|>\varepsilon\right)$$
$$\leqslant CS_{nj_k}^{-4}S_{nN}^2\leqslant CS_{nj_k}^{-4}S_{nj_{k+1}}^2\leqslant C2^{-k} \tag{2.17}$$

and

$$P\left(|S_{nj_k}^{-2}\max_{j_k<l\leqslant j_k+1}\sum_{j=n+j_k+1}^{n+l}\left(\sum_{i=n+1}^{n+N}\omega_{nj}(t_i)\bar{u}_n(v_i)\right)(\varepsilon_i-E\varepsilon_i)|>\varepsilon\right)\leqslant C2^{-k}. \tag{2.18}$$

Combining (2.15), (2.16), (2.17) and (2.18),Borel–Cantelli Lemma and using the standard subsequence method, when $N \to \infty$, we have

$$S_{nN}^{-2} \sum_{i=n+1}^{n+N} \bar{u}_n(v_i)(\varepsilon_j - E\varepsilon_j) \to 0, a.s. \tag{2.19}$$

$$S_{nN}^{-2} \sum_{j=n+1}^{n+N} \left(\sum_{i=n+1}^{n+N} \omega_{nj}(t_i) \bar{u}_n(v_i) \right) (\varepsilon_i - E\varepsilon_i) \to 0, a.s. \tag{2.20}$$

Combining (2.19) and (2.20), we have

$$I_1 \to 0, \ a.s.n, \ N \to \infty. \tag{2.21}$$

Notice that

$$\begin{aligned}\sum_{j=n+1}^{n+N} \left| x_j - u_n(v_j) \right| &= \sum_{j=n+1}^{n+N} \left| \sum_{i=1}^{n} (x_j - x_i) \frac{K_1((v_i - v_j)/b_n)}{\sum_{i=1}^{n} K_1((v_i - v_j)/b_n)} \right| \\ &= \sum_{j=n+1}^{n+N} \left| \sum_{i=1}^{n} (x_j - x_i) \right. \\ &\quad \times \left. \frac{K_1((v_i - v_j)/b_n) \left\{ I(\|v_i - v_j\| \geqslant b_n^{1/2}) + I(\|v_i - v_j\| < b_n^{1/2}) \right\}}{\sum_{i=1}^{n} K_1((v_i - v_j)/b_n)} \right| \\ &\leqslant \sum_{j=n+1}^{n+N} \sum_{i=1}^{n} |x_j - x_i| \frac{o(1)}{Cn} + \sum_{j=n+1}^{n+N} \sum_{i=1}^{n} |x_j - x_i| \frac{C}{n} I\left(|x_j - x_i| < b_n^{1/2} \right).\end{aligned} \tag{2.22}$$

It follows that

$$\begin{aligned} I_2 &\leqslant \max_j \left| \bar{u}_n(v_j) - \sum_{i=n+1}^{n+N} \omega_{nj}(t_i) \bar{u}_n(v_i) || \beta | S_{nN}^{-2} \sum_{j=n+1}^{n+N} |x_j - u_n(v_j)| \right| \\ &\leqslant C S_{nN}^{-2} \frac{o(1)}{Cn} \left(\sum_{i=n+1}^{n+N} [n|x_j| + \sum_{i=1}^{n} |x_i|] \right) + S_{nN}^{-2} C N b_n^{1/2} = o(1). \end{aligned} \tag{2.23}$$

(2.12), (2.21), (2.23) and (2.11) together imply (2.9) of Theorem 2. Observe that

$$\begin{aligned} \tilde{g}_{nN}(t) - g(t) &= \beta - \beta_{nN} g_{2N}(t) + \left(\sum_{j=n+1}^{n+N} \omega_{nj}(t) g(t_j) - g(t) \right) + \sum_{j=n+1}^{n+N} \omega_{nj}(t) \varepsilon_j \\ &=: A_{nN}(t) + B_{nN}(t) + C_{nN}(t). \end{aligned} \tag{2.24}$$

By (2.9) and $\sup_t g_{2N}(t) \leqslant C$, it follows that

$$\sup_{0\leqslant t\leqslant 1} A_{nN}(t) \to 0, a.s.n \to \infty.$$

Using the similar reasoning as the proof of (2.12), we have

$$\sup_{0\leqslant t\leqslant 1} B_{nN}(t) \to 0, a.s.n \to \infty.$$

Noting that

$$E\left(\max_{1<k\leqslant N} \sum_{j=n+1}^{n+k} \omega_{nj}(t)(\varepsilon_j - E\varepsilon_j)\right)^2 \leqslant C \sum_{j=n+1}^{n+N} \omega_{nj}^2(t)$$
$$\leqslant C \max_{t,j} |\omega_{nj}(t)| \max_t \sum_{j=n+1}^{n+N} |\omega_{nj}(t)| \leqslant CN^{-1}. \tag{2.25}$$

Applying (2.25), the same arguments as in the proof of that (2.13) implies (2.19), $E\varepsilon_j = (x_j - u_n(v_j))\beta$ and (2.22) together prove that

$$\sup_{0\leqslant t\leqslant 1} C_{nN}(t) \to 0, a.s.n \to \infty.$$

Hence, Theorem 2 is proved.

Because ρ^--mixing sequence is more general than negatively associated (NA) sequence or ρ^*-mixing sequence. So we have the following four Corollaries.

Corollary 1. Let $\{e_i, i \geqslant 1\}$ be a negatively associated (NA) sequence, in addition to (2.6), assumptions that (A1)–(A4), (A6) hold, then

$$E|\beta_{nN} - \beta|^2 = O(S_{nN}^{-2}) \tag{2.26}$$

$$E\left(\sup_{t\in[0,1]} |\tilde{g}_{nN}(t) - g(t)|^2\right) = O(S_{nN}^{-2}) + O(N^{-1}) \tag{2.27}$$

Corollary 2. Let $\{e_i, i \geqslant 1\}$ be a negatively associated (NA) sequence, in addition to (2.6), assumptions that (A1)–(A6), then

$$\lim_{n,N\to\infty} \beta_{nN} = \beta a.s. \tag{2.28}$$

$$\lim_{n,N\to\infty} \sup_{t\in[0,1]} |\tilde{g}_{nN}(t) - g(t)| = 0 a.s. \tag{2.29}$$

Corollary 3. Let $\{e_i, i \geqslant 1\}$ be a ρ^*-mixing sequence, in addition to (2.6), assumptions that (A1)–(A4), (A6) hold, then

$$E|\beta_{nN} - \beta|^2 = O(S_{nN}^{-2}) \tag{2.30}$$

$$E\left(\sup_{t\in[0,1]} |\tilde{g}_{nN}(t) - g(t)|^2\right) = O(S_{nN}^{-2}) + O(N^{-1}) \tag{2.31}$$

Corollary 4. Let $\{e_i, i \geqslant 1\}$ be a ρ^*-mixing sequence, in addition to (2.6), assumptions that (A1)–(A6), then

$$\lim_{n,N\to\infty} \beta_{nN} = \beta a.s. \tag{2.32}$$

$$\lim_{n,N\to\infty} \sup_{t\in[0,1]} |\tilde{g}_{nN}(t) - g(t)| = 0 a.s. \tag{2.33}$$

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